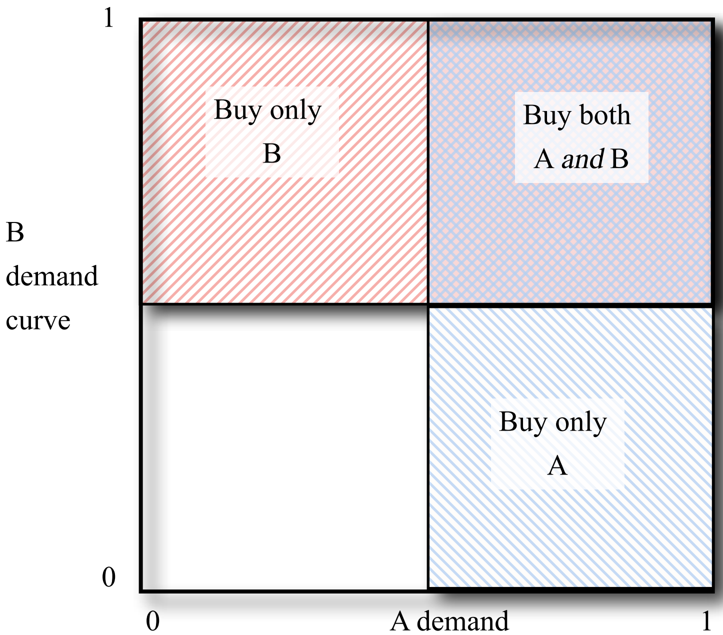
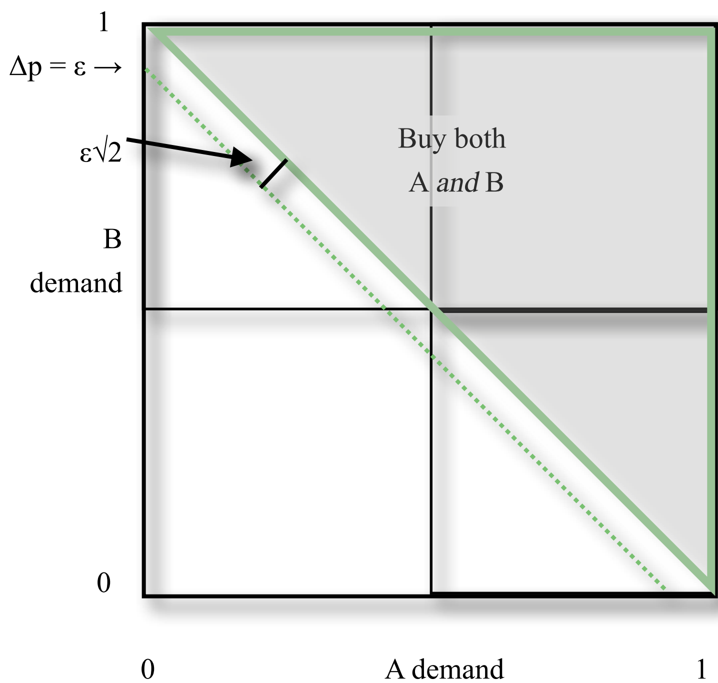
# ***Bundling***

*Economics 243* | *Fall 2018*




In the base monopoly case with demand curves p = 1 - q, the two goods are priced independently at p = ½ so only consumers in the upper half of the demand curves for the two goods purchase it. Half of consumers purchase each good. When demand is uncorrelated, the overlap is for consumers who buy both. Total revenue is TR = 2pq = 2 x ½ x ½ = ½ = π (since we assumed FC = MC = 0).

What happens if we lower price by ε? Since we are at the optimum, we know profit falls. Specifically, we add to volume (Q = 1 + 2ε), but we lose on price (p = ½ - ε) which multiplied out is $\pi = ½ - 2\varepsilon^2$.




If we bundle A and B, and charge price p = 1 then we pick up that marginal consumers who are just willing to buy good A and good B, and we pick up all consumers willing to pay p = ½ (or more) for each. In other words, we sell to all consumers in the gray area, half the square for total revenue = π = ½. But we can do better. If we lower price by ε we pick up a strip that is ε√2 wide (the diagonal of a 45º right triangle) and √2 long. So we lose approximately ½ε from the lower price but pick up ε√2 = 1.41ε in volume. Profits rise.

The calculations for the optimal price are below.

In our case total sales for bundle price p < 1 will be the area of the square less that of the lower left triangle. So we have $TR = \pi = p(1 - ½ p^2) = p - ½ p^3$. Take the first derivative to maximize and we need to set $1 = 3/2\ p^2$ or $p = \sqrt{2/3} \approx 0.82$. In general pure bundling beats separate selling; however, putting in marginal costs results in complex (non-linear) calculations.

More generally, we have two regions. One is for a bundle price below p=1, in which case total revenue is the area of the square less than that of the triangle, or $Q=1-\frac{1}{2}p^2$, half of base times height. For prices greater than p=1, total revenue is the area of the triangle, which has a side of length 2-p or $Q=(2-p)^2/2$. To find MR we need to solve for p=f(q) and then get TR = pq and find $\partial p/\partial q$. We then have after suitable simplification:

$$p \leq 1 \rightarrow q = 1-\tfrac{1}{2}p^2 \text{ so then } p = 2^{1/2}(1-q)^{1/2}$$

$$\text{and TR} = q2^{1/2}(1-q)^{1/2}$$

$$\text{thus MR} = p - q[2^{-1/2}(1-q)^{-1/2}] = p - q/p = 3/2\ p - 1/p.$$

$$p \geq 1 \rightarrow q = (2-p)^2/2 \text{ so then } p = 2 - (2q)^{1/2}$$

$$\text{and TR} = 2q - (2q^3)^{1/2}$$

$$\text{thus MR} = 3/2p - 1.$$

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The relationship is **<u>not</u>** linear. If we plot profits as a function of marginal costs, then when marginal costs are low, so that we're close to our baseline solution for MC=0, bundling remains profitable. Profits do fall, as they do for a (monopolist) selling goods that aren't bundled. It turns out that the profitability for bundling falls more quickly, so that **when MC is high** (approaching 2) **bundling is less effective than selling independently**. The break point with our arithmetically simple $p_i = 1- q_i$ demand model is at MC ≈ 0.28 where (optimally) the price of the bundle is $p^{bundle} \approx 0.91$ and where the price of an unbundled good is $p^{unbundled} \approx 0.57$. At that level of MC, profits $\pi^{bundle} = \pi^{unbundled} \approx 0.37$ (vs $\pi = 0.5$ for a monopoly and $\pi=0.54$ for a pure bundle when MC=0).

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**What happens if we add the option to buy the goods separately?**

- The elasticity of demand for goods by themselves is lower, because part of the demand is picked up by the bundle. Lerner's Rule implies that $1/\varepsilon$ is thus higher so price must be higher.
- Again the elasticity of demand for the bundle is lower, because part of the demand is picked up by sales of individual goods. Price for the bundle is thus also higher.

⇒ hence net profits are higher. In our example with MC = 0 we get the price for the bundle as ≈ 0.86 (versus 0.82) and $\pi \approx 0.549$ (versus $\pi \approx 0.544$).

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**How about the case where demand is correlated?**

If the correlation is **negative**, then bundling always increases profits relative to selling separately.

If it's **positive**, then selling one tends to increase demand for the other, so that consumers choose both on their own. Bundling then picks up far fewer sales. So above a certain threshold it becomes **less** profitable to bundle than to sell separately.

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**How about moving to more than two goods?** That generally increases the benefits of bundling, as it increases the additional sales picked up by lowering the bundle price. The logic is that instead of just picking up consumers who value the two goods a bit under the separate-sale (monopoly) price, you pick up consumers who value 2 of 3, or 4 of 7… The more goods bundled, the better bundling becomes.